GLOBAL HIGHER REGULARITY OF SOLUTIONS TO $p(x,t)$-PARABOLIC EQUATIONS

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This report is devoted to the class of singular parabolic equations with variable nonlinearity which involve the $p(x,t)$-Laplacian. The homogeneous Cauchy-Dirichlet problem is considered

$$\begin{cases} u_t = \operatorname{div}\left(|\nabla u|^{p(z)-2}\nabla u\right) + f(z,u) \quad \text{in } Q_T = \Omega \times (0,T), \\ u(x,0) = u_0(x) \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega \times (0,T) \end{cases} \tag{0.1}$$

where $z = (x,t) \in Q_T$, $\Omega \subset \mathbb{R}^d$, $d \geq 2$, is a bounded domain, $T < \infty$, $p(z) : Q_T \mapsto \mathbb{R}$ is a given function. Equations of the type (0.1) fall into the class of equations with nonstandard growth, or variable nonlinearity. These equations have been intensively studied in the recent years. Equation (0.1) degenerates or becomes singular at the points where $|\nabla u| = 0$. There already exists a wide literature on the issues of existence and uniqueness of **weak solutions** of problem (0.1), see, e.g., [1].

We are interested in the sufficient conditions of existence of **strong solutions** to problem (0.1) and in the conditions on the data that provide **the global higher regularity** of such solutions. Namely, the existence of the derivatives $u_t \in L^2(Q_T)$ and $D^2_{x_i x_j} u$ as elements of the Lebesgue spaces $L^{q(\cdot)}(Q_T)$ with possibly variable exponent q. In comparison with [2], **the global higher regularity results** have been proved for **strong solutions** of problem (0.1):

$$\sup_{(0,T)} \|\nabla u(t)\|^2_{2,\Omega} + \|u_t\|^2_{2,Q_T} + \int_{Q_T} \left(|\nabla u|^{p(z)-2}|u_{xx}|^2 + |u_{xx}|^{p(z)}\right) dz \leq C\|\nabla u_0\|^2_{2,\Omega} + C_f.$$

This report has been prepared in cooperation with S. Antontsev and S. Shmarev.

References

[1] S. Antontsev and S. Shmarev, Evolution PDEs with nonstandard growth conditions, Atlantis Studies in Differential Equations, V.4, Atlantis Press, Paris, 2015.

[2] S. Antontsev and S. Shmarev, Higher regularity of solutions of singular parabolic equations with variable nonlinearity, Applicable Analysis (2017), accepted, http://dx.doi.org/10.2991/978-94-6239-112-3